

Reg. No.

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Question Paper Code : 27661

B.Arch. DEGREE EXAMINATION, DECEMBER 2015/JANUARY 2016

First Semester

MA 6153 – MATHEMATICS

(Regulation - 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Write down the expansions of $\sin x$ and $\cos x$.
2. Find the area of a triangle which has each side equal to 4 cm.
3. Find the equation of a straight line that passes through the point $(1, -1, 2)$ and perpendicular to the plane $2x - 3y - 5z = 1$.
4. Find the equation of the sphere whose end points of the diameter are $(2, -1, 4)$ and $(-2, 2, -2)$.
5. Obtain the value of the integral $\int_1^2 \frac{dx}{x^2 + 5x + 6}$.
6. Evaluate $\int_0^{\pi/2} \cos^6 x \, dx$.
7. Solve $(D^2 + D + 1)y = 0$.

8. Transform the equation $xy'' + y' + \frac{y}{x} = \log x$ into a second order differential equation with constant coefficients.
9. Find the median of the set of numbers 10, 6, 5, 25, 15, 18, 20, and 37.
10. A coin and a dice are tossed. What is the probability of getting a head and a '1' ?

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Prove that $\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$ using DeMoivre's theorem. (8)

- (ii) Express $\tan 6\theta$ in terms of $\tan \theta$. (8)

OR

- (b) (i) The radii of two circles are 3 cm and 4 cm respectively. Find the ratio of their areas. (4)

- (ii) From a solid right circular cylinder with height 10 cm and base radius 6 cm, a right circular cone of the same height and base is removed. Find the volume of the remaining solid. (6)

- (iii) Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboid to the sum of the surface areas of the three cubes. (6)

12. (a) (i) Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{-3}$ are coplanar. Also find the equation of the plane in which they lie. (8)

- (ii) Find the equation of the plane through the line $\frac{x-1}{3} = \frac{y-4}{2} = \frac{z-4}{-2}$ and parallel to the line $\frac{x+1}{2} = \frac{y-1}{-4} = \frac{z+2}{1}$. Hence find the shortest distance between them. (8)

OR

- (b) (i) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, x + y + z = 3$ as a great circle. (8)
- (ii) Find the equations of the spheres passing through the circle $x^2 + y^2 + z^2 - 6x - 2z + 5 = 0, y = 0$ and with tangent plane $3y + 4z + 5 = 0$. (8)

13. (a) (i) Evaluate $\int_0^1 x^4 (1-x^2)^{\frac{3}{2}} dx$. (8)

(ii) Derive the reduction formulae for $\int \tan^n x dx$ and $\int \sec^n x dx$. (8)

OR

- (b) (i) Expand $e^x \cos y$ in powers of x and y upto third degree terms. (8)
- (ii) Examine the extrema of $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$. (8)

14. (a) (i) Solve $(D^2 + 5D + 4)y = e^{-x} \sin 2x$. (8)
- (ii) Solve the simultaneous equations $Dx - (D - 2)y = \cos 2t; (D - 2)x + Dy = \sin 2t$. (8)

OR

- (b) (i) Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x + 2$. (8)
- (ii) Solve $\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = 12 \frac{\log x}{x^2}$. (8)

15. (a) (i) Calculate the mean and standard deviation of the following frequency distribution : (8)

Weekly wages in `	4.5-12.5	12.5-20.5	20.5-28.5	28.5-36.5	36.5-44.5	44.5-52.5	52.5-60.5	60.5-68.5	68.5-76.5
No. of men	4	24	21	18	5	3	5	8	2

- (ii) In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Find the mean of x , mean of y and the coefficient of correlation between x and y . (8)

OR

- (b) (i) A bag contains 40 tickets numbered 1, 2, 3 ..., 40 of which four are drawn at random and arranged in ascending order ($t_1 < t_2 < t_3 < t_4$). Find the probability of t_3 being 25. (8)
- (ii) Given that $P(A) = 1/4$, $P(B) = 1/3$ and $P(A \cup B) = 1/2$, evaluate $P(A/B)$, $P(B/A)$, $P(A \cap B')$ and $P(A/B')$ (8)

Reg. No. :

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Question Paper Code : 37011

B. Arch. DEGREE EXAMINATION, JANUARY 2014.

First Semester

MA 6153 — MATHEMATICS

(Regulation 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sinh x - \sin x}{x^3} \right)$.
2. Find the real part of $\sin(x + iy)$.
3. What do you mean by direction cosines and direction ratios of a line?
4. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line.
5. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}} dx$.
6. Prove that $\int_0^a f(x) dx = \int_0^a f(a - x) dx$.
7. Solve $\frac{d^2x}{dt^2} + 3a \frac{dx}{dt} - 4a^2x = 0$.
8. Find the particular integral of $(D^2 - 5D + 6)y = 2^x$.
9. What do you mean by correlation?
10. Prove that for any event A , $P(\bar{A}) = 1 - P(A)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If $2 \cos \theta = x + \frac{1}{x}$, then prove that $2 \cos r\theta = x^r + \frac{1}{x^r}$. (8)

(ii) Express $\cos 6\theta$ in series of powers of $\cos \theta$. (8)

Or

(b) (i) Find the volume of a sphere of radius 'a' units. (8)

(ii) If $\tanh 3y = \tanh 3x$, then show that $\tanh y = \frac{we^x - e^{-x}}{we^x + e^{-x}}$. (8)

12. (a) (i) Find the equation of the plane which passes through the point $(3, -3, 1)$ and is perpendicular to the planes $7x + y + 2z = 6$ and $3x + 5y - 6z = 8$. (8)

(ii) Prove that the points $(3, 2, 4)$, $(4, 5, 2)$ and $(5, 8, 0)$ are collinear. (8)

Or

(b) (i) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. (8)

(ii) Find the equation of the sphere which touches the planes $x - 2y - 2z = 7$ at the point $(3, -1, -1)$ and passes through the point $(1, 1, -3)$. (8)

13. (a) (i) Evaluate $\int_0^{\frac{\pi}{6}} \cos^4 3\theta \sin^3 6\theta d\theta$ and $\int_0^{\pi/2} \cos^9 x dx$. (8)

(ii) Expand $f(x, y) = e^y \log(1+x)$ in powers of x and y . (8)

Or

(b) Find the maximum and minimum values of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. (16)

14. (a) (i) Solve $(D^2 + 4D + 3)y = e^{-x} \sin x + xe^{3x}$. (8)

(ii) Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$. (8)

Or

(b) Solve the simultaneous equations :

$$\frac{dx}{dt} + 5x - 2y = t; \quad \frac{dy}{dt} + 2x + y = 0. \quad (16)$$

15. (a) Given below is the distribution of 140 candidates obtaining marks X or higher in a certain examination (all marks are given in whole numbers) :

X :	10	20	30	40	50	60	70	80	90	100
More than c.f. :	140	133	118	100	75	45	25	9	2	0

Calculate the mean, median and mode of the distribution. (16)

Or

(b) (i) State and prove the addition theorem of probability. (8)

(ii) If A and B are two independent events then prove that \bar{A} and \bar{B} are also independent. (8)

Reg. No.

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Question Paper Code : 57498

B.Arch. DEGREE EXAMINATION, MAY/JUNE 2016

First Semester

MA 6153 – MATHEMATICS

(Regulation 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Separate $e^{i\theta}$ into real and imaginary parts.
2. Show that $\sin 1 - \cos 1 = \frac{2}{3!} - \frac{4}{5!} + \frac{6}{7!} \dots\dots$
3. Find the projection of the segment joining the points (1, 2, 3), (6, 7, 9) on the line whose direction ratios are (1, 2, -3).
4. Find the angle between the lines $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-1}{-1}$ and $\frac{x+2}{1} = \frac{y+3}{-7} = \frac{z+4}{2}$.
5. Prove that $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$
6. Obtain the stationery points of the function $f(x, y) = x^3 + y^3 - 3axy$
7. Solve the differential equation $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 13y = 0$.

8. Reduce $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$ to a homogeneous equation.
9. Calculate mean if median = 4 and mode = 2.
10. A card is drawn from a well shuffled pack of playing cards. What is the probability that it is either diamond or king ?

PART - B (5 × 16 = 80 Marks)

11. (a) (i) Show that $\left(\frac{\cos \theta + i \sin \theta}{\sin \theta + i \cos \theta}\right)^4 = \cos 8\theta + i \sin 8\theta$.

(ii) If $\sin(A + iB) = x + iy$ prove that

(1) $\frac{x^2}{\cos^2 A} + \frac{y^2}{\sin^2 A} = 1$ and

(2) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

OR

(b) (i) Express $\cos 6\theta$ in terms of multiples $\cos \theta$.

(ii) If $x + iy = \tan(A + iB)$ prove that $x^2 + y^2 + 2x \cot 2A = 1$.

12. (a) (i) Find the points on the lines $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-3}{-3}$ and $\frac{x-2}{-2} = \frac{y-1}{3} = \frac{z+1}{5}$ which are nearer to each other.

(ii) Find the centre and radius of the circle $x^2 + y^2 + z^2 = 9, x + y + z = 3$.

OR

(b) (i) Find the equation of the sphere passing through the points (1, 5, -1), (4, -1, 2), (0, -2, 3) and (2, 0, 1).

(ii) Show that the lines $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ and $\frac{x-1}{5} = \frac{y+2}{-4} = \frac{z-3}{5}$ are perpendicular.

13. (a) (i) Expand $x^2y^2 + 2x^2y + 3xy^2$ as a Taylor's series about $x = -2$ and $y = 1$.

(ii) Evaluate $\int_0^1 \frac{4x^2 + 3}{8x^2 + 4x + 5} dx$.

OR

(b) (i) If $I_n = \int_0^{\pi/2} x^n \sin x dx$ and $n > 1$, show that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$.

(ii) Show that $\int_0^{\pi} \theta \sin^6 \theta \cos^4 \theta d\theta = \frac{3}{512} \pi^2$.

14. (a) (i) Obtain the solution to $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 2x^2 - x + 3$.

(ii) Solve : $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \sin(\log x^2)$

OR

(b) (i) Solve the simultaneous equations $Dx + y = \sin t$, $x + Dy = \cos t$ given that $x = 2$, $y = 0$ at $t = 0$.

(ii) Solve the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x} \sin x$.

15. (a) (i) Calculate the standard deviation for the following distribution :

x	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f	1	5	12	22	17	9	4

(ii) The two regression lines are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. Find the correlation coefficient.

OR

- (b) (i) A bag contains 30 balls numbered from 1 to 30. One ball is drawn at random. Find the probability that the number of the drawn ball will be a multiple of 5 or 9.
- (ii) Find the probability of drawing a queen, a king and a knave in that order from a pack of cards in three consecutive draws, the cards drawn not being replaced.

Reg. No. :

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Question Paper Code : 72064

B.Arch. DEGREE EXAMINATION, APRIL/MAY 2017

First Semester

MA 6153 — MATHEMATICS

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State DeMoivre's theorem.
2. The major and minor axis of an ellipse are 15.0 cm and 9.0 cm respectively. Find the approximate perimeter.
3. Find the equation of the plane which passes through the point (3, -3, 1) and parallel to the plane $2x + 3y + 5z + 6 = 0$.
4. Find the equation of the sphere having the points (-4, 5, 1) and (4, 1, 7) as ends of a diameter.
5. Compute the value of the integral $\int_0^{\pi/2} \sin^6 x \, dx$.
6. Prove that $\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$.
7. Solve $(D^2 + 5D + 6)y = 0$.
8. Find the particular integral of $(D^2 + 6D + 9)y = e^{-3x}$.
9. Write merits and demerits of a mode of the frequency distribution.
10. Define mutually exclusive events with an example.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $\left(\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i \sin\left(\frac{n\pi}{2} - n\theta\right)$. (8)

(ii) If $\sin \alpha + \sin \beta + \sin \gamma = \cos \alpha + \cos \beta + \cos \gamma = 0$, then prove that $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$. (8)

Or

(b) (i) Determine the area of a regular hexagon which has sides 8 cm long. (8)

(ii) A boiler consists of a cylindrical section of length 8m and diameter 6 m, on one end of which surmounted a hemispherical section of diameter 6 m and on the other end a conical section of height 4m and base diameter 6 m. Calculate the volume of the boiler and the total surface area. (8)

12. (a) (i) A variable plane at a constant distance p from the origin meets the axes in A, B, C . Planes are drawn through A, B, C parallel to the coordinate planes. Show that the locus of their point of intersection is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$. (8)

(ii) Find, in symmetrical form, the equations of the line $x + y + z + 1 = 0$, $4x + y - 2z + 2 = 0$. (8)

Or

(b) (i) Find the magnitude and the equations of the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$. (8)

(ii) Find the equations of the spheres passing through the circle $x^2 + y^2 + z^2 - 6x - 2z + 5 = 0$, $y = 0$ and touching the plane $3y + 4z + 5 = 0$. (8)

13. (a) (i) Evaluate $\int \frac{x+4}{6x-7-x^2} dx$. (8)

(ii) Expand $e^{\sin x}$ by Taylor's series in power of x up to the terms containing x^4 . (8)

Or

(b) (i) Derive the reduction formula for $\int \sin^n x dx$. (8)

(ii) Find the maximum and minimum values of $3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the interval $(0, 2)$. (8)

14. (a) (i) Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x)$. (8)

(ii) Solve $(x^2 D^2 - xD + 1)y = \log x$. (8)

Or

(b) (i) Solve $(D^2 + 2D + 2)y = e^{-x} \sin x$. (8)

(ii) Solve the simultaneous equations $\frac{dx}{dt} + 2y + \sin t = 0$,

$\frac{dy}{dt} - 2x - \cos t = 0$, given that $x = 0$ and $y = 1$ when $t = 0$. (8)

15. (a) (i) Calculate the mean and standard deviation for the following frequency distribution: (8)

x : 8.5 16.5 24.5 32.5 40.5 48.5 56.5 64.5 72.5

f : 4 24 21 18 5 3 5 8 2

(ii) State and prove the addition law of probability. (8)

Or

(b) (i) Find the correlation coefficients for the following data: (8)

x : 105 104 102 101 100 99 98 96 93 92

f : 101 103 100 98 95 96 104 92 97 94

(ii) A pair of dice is tossed twice. Find the probability of scoring 7 points (1) once, (2) at least once (3) twice. (8)

Reg. No. :

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Question Paper Code : 77187

B.Arch. DEGREE EXAMINATION, APRIL/MAY 2015.

First Semester

MA 6153 — MATHEMATICS

(Regulation 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the value of $\sinh\left(\frac{\pi}{2}\right)$.
2. Evaluate $\iint_{\mathbf{R}} xy dx dy$ where \mathbf{R} is the area between the line $y = x + 1$ and the curve $y = x^2 + 1$.
3. Find the direction cosines of the lines joining the following points : $(5, -2, 7)$, $(2, 3, 5)$.
4. The normal to a plane has the direction ratios 3, 4, 12. If the plane passes through $(1, -3, 1)$, find its equation.
5. Evaluate $\int \frac{dx}{x^2 \sqrt{1+x^2}}$.
6. Evaluate $\int_{-1}^1 \frac{xe^{x^2}}{1+x^2} dx$.
7. Find the particular integral of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$.

8. Reduce $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$ to an equation with constant coefficients.
9. There are 250 observations in a group. The average of first 100 is 5 and the average of remaining 150 is $8\frac{1}{3}$. Find the average of the whole group.
10. Let A and B be independent events such that $P(A) = 0.5$ and $P(B) = 0.8$. Find $P(\bar{A} \cap \bar{B})$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find all the values of $\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^{3/4}$ and show that the continued product of all the values is 1. (8)
- (ii) Find the volume of the solid bounded by the planes $z = 0$, $x = 0$, $y = 0$, $x^2 + y^2 = 4$ and $z = 6 - xy$ for $x \geq 0$, $y \geq 0$, $z \geq 0$. (8)

Or

- (b) (i) Express $\sin^8 \theta$ in a series of cosines of multiples of θ . (6)
- (ii) Determine the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, $z = x$, $z = 2$ and $y = 4 - x^2$ in the first octant. (5)
- (iii) Evaluate $\iint_{\mathbf{R}} (x^2 + y^2) dx dy$ where \mathbf{R} is the region of the xy -plane bounded by $y = x^2$, $x = 2$ and $y = 1$. (5)
12. (a) (i) Show that the lines $\frac{x-7}{2} = \frac{7-10}{3} = \frac{z-13}{4}$ and $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-7}{3}$ are coplanar. Find the equation of the plane of coplanarity. (8)
- (ii) Find the centre and radius of the circle $x^2 + y^2 + z^2 - 2x - 4y - 6z = 2$, $x + 2y + 2z = 20$. (8)

Or

(b) (i) Find the shortest distance between the lines $\frac{x-5}{3} = \frac{y-6}{-4} = \frac{z-9}{1}$ and $2x-2y+z=3$, $2x-y+2z=9$. (8)

(ii) Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as a great circle. (8)

13. (a) (i) Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$. (8)

(ii) Use Taylor's series to expand $f(x,y)=x^2+xy+y^2$ in powers of $(x-1)$ and $(y-2)$ upto second degree. (8)

Or

(b) (i) Compute $\int_0^{\pi/2} \cos^2 x \sin^4 x dx$. (4)

(ii) Evaluate $\int_0^{\pi/2} x^2 \sqrt{1-x^2} dx$. (4)

(iii) Find the relative extrema of the function

$$f(x,y) = 4y^3 + x^2 - 12y^2 - 36y + 2. \quad (8)$$

14. (a) (i) Solve $\frac{d^2y}{dx^2} + y = x \sinh x$. (8)

(ii) Solve the following simultaneous equations :

$$\frac{dx}{dt} + 2x - 3y = t, \quad \frac{dy}{dt} + 2y - 3x = e^{2t}. \quad (8)$$

Or

(b) (i) Solve $(D^2 + 4D + 3)y = e^{-x} \sin x + xe^{-3x}$. (8)

(ii) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = x^2 \cos(\log x)$. (8)

15. (a) (i) Calculate the coefficient of correlation and obtain the least square regression line of y on x for the following data : (8)

x : 1 2 3 4 5 6 7 8 9
 y : 9 8 10 12 11 13 14 16 15

- (ii) A certain carton of eggs has 3 bad and 9 good eggs.
- (1) If an omelette is made of 3 eggs randomly chosen from the carton, what is the probability that there are no bad eggs in the omelette?
 - (2) What is the probability of having atleast 1 bad egg in the omelette?
 - (3) What is the probability of having exactly 2 bad eggs in the omelette? (8)

Or

- (b) (i) The first of the two samples has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 156 and standard deviation $\sqrt{13.44}$, find the standard deviation of the second group. (8)
- (ii) Let A and B be two events such that $P(A)=0.36$ and $P(A \cup B)=0.64$, find $P(B)$ if
- (1) A and B are mutually exclusive
 - (2) A and B are independent. (8)

Reg. No. :

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Question Paper Code : 80605

B.Arch. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016

First Semester

MA 6153 — MATHEMATICS

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write a formula for $\sin \theta$ and $\cos \theta$ in terms of an exponential function.
2. Find the area of the circle having a circumference of 70mm.
3. Find the angle between the two lines whose direction ratios are (a, b, c) and (l, m, n) .
4. Find the centre and radius of the sphere
 $6(x^2 + y^2 + z^2) - 8x + 10y - 18z - 13 = 0$.
5. Compute the value of the integral $\int_0^{\pi/2} \cos^6 x \, dx$.
6. Write the condition for $f(x)$ is maximum at $x = a$.
7. Solve : $(D^2 + 6D + 9)y = 0$.
8. Find the particular integral of $(D^2 + 5D + 6)y = e^{-2x}$.
9. How mean, median and mode of a frequency distribution are related?
10. Give the axiomatic definition of probability.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that

$$(1 + \cos \theta + i \sin \theta)^n + (1 + \cos \theta - i \sin \theta)^n = 2^{n+1} \cos^n(\theta/2) \cos(n\theta/2) \quad (8)$$

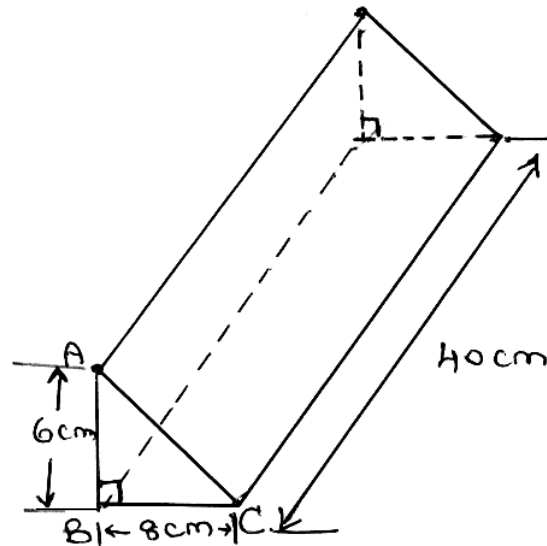
- (ii) If
- $\sin \alpha + \sin \beta + \sin \gamma = \cos \alpha + \cos \beta + \cos \gamma = 0$
- , prove that
- $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$
- . (8)

Or

- (b) (i) A copper wire, when bent in the form of a square, encloses an area of 484 cm
- ²
- . If the same wire is bent in the form of a circle, find the area enclosed by it. (8)

- (ii) Calculate the volume of the right-angled triangular prism shown in the following figure.

Also determine its total surface area. (8)



12. (a) (i) Find the equation of the plane through the line of intersection of the planes
- $x + y + z = 1$
- and
- $2x + 3y + 4z = 5$
- which is perpendicular to the plane
- $x - y + z = 0$
- . (8)

- (ii) Show that the lines
- $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$
- ;
- $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$
- are coplanar. Find their common point and the equation of the plane in which they lie. (8)

Or

- (b) (i) Find the points on the lines $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$; $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ which are nearest to each other. Find the shortest distance between the lines and its equations. (8)
- (ii) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle. (8)
13. (a) (i) Evaluate $\int \frac{6x+5}{\sqrt{6+x-2x^2}} dx$. (8)
- (ii) Show that $(1 + \cos x) \sin x$ is a maximum when $x = \pi/3$. (8)
- Or
- (b) (i) Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$. (8)
- (ii) Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1.1$ correct to 4 decimal places. (8)
14. (a) (i) Solve : $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x$. (8)
- (ii) Solve : $(x^2 D^2 - xD + 1)y = x^2$. (8)
- Or
- (b) (i) Solve : $(D^2 - 5D + 6)y = e^x \cos 2x$. (8)
- (ii) Solve the simultaneous equations $\frac{dx}{dt} - y = t$, $\frac{dy}{dt} + x = t^2$ given that $x = 0$ and $y = 1$ when $t = 0$. (8)
15. (a) (i) Calculate the mean and standard deviation for the following frequency distribution : (8)
- | | | | | | | | |
|-------|---|---|---|----|----|----|----|
| x : | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| f : | 3 | 6 | 9 | 13 | 8 | 5 | 4 |
- (ii) A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chance of winning. (8)

Or

- (b) (i) In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate \bar{x} , \bar{y} and the coefficient of correlation between x and y . (8)
- (ii) If A and B are independent events, then prove that their compliments are also independent events. (8)
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